

# Electronic transport through ballistic chaotic cavities: Reflection symmetry, direct processes, and symmetry breaking

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We extend previous studies on transport through ballistic chaotic cavities with spatial left-right (LR) reflection symmetry to include the presence of direct processes. We first analyze fully LR-symmetric systems in the presence of direct processes and compare the distribution  $w(T)$  of the transmission coefficient  $T$  with that for an asymmetric cavity with the same “optical”  $S$  matrix. We then study the problem of “external mixing” of the symmetry caused by an asymmetric coupling of the cavity to the outside. We first consider the case where symmetry breaking arises because two symmetrically positioned waveguides are coupled to the cavity by means of asymmetric tunnel barriers. Although this system is asymmetric with respect to the LR operation, there is an effect of the symmetry of the cavity it was constructed from. Second, we break LR symmetry in the absence of direct processes by asymmetrically positioning the two waveguides and compare the results with those for the completely asymmetric case.

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## I. INTRODUCTION

The problem of chaotic wave scattering is of great interest in various branches of physics, such as optics, nuclear, mesoscopic, and microwave physics. The study of quantum-mechanical scattering problems whose classical dynamics is chaotic has been further motivated by recent experiments on quantum-electronic transport in microstructures consisting of a cavity connected to leads [1]. We know that symmetries have very interesting effects on the properties of the electric conductance in mesoscopic systems: time-reversal and spin-rotational symmetries [2,3], as well as spatial-reflection symmetries [4,5] have been studied in the literature.

The problem of electronic transport through asymmetric (AS) chaotic cavities is addressed in detail in Ref. [6] in an independent-electron approximation. In that reference, the possibility of direct processes due to the presence of short paths is accounted for by specifying the average, or optical,  $S$  matrix  $\langle S \rangle$  within an information-theoretic approach. The statistical distribution for the  $S$  matrix is known as Poisson’s kernel, in which  $\langle S \rangle$  is a parameter. When  $\langle S \rangle = 0$ , i.e., in the absence of direct processes, the statistical distribution reduces to the invariant measure for the appropriate universality class.

Microstructures with reflection symmetry and a chaotic classical dynamics are studied in Refs. [4] and [5]. The analysis is performed in the absence of direct processes, so that the statistical distribution of the  $S$  matrix is the invariant measure for the universality class in question and the relevant spatial symmetry: the latter is a symmetry of the *full* system under consideration, i.e., the cavity plus the two leads

that connect the cavity to the outside.

One purpose of the present paper is to extend the study of Refs. [4] and [5] to include the presence of direct processes. We consider two-dimensional systems with spinless particles and concentrate on left-right (LR) symmetry only, i.e., symmetry under reflection through an axis perpendicular to the current. We also restrict the analysis to time-reversal-invariant (TRI) problems. One particular way of inducing direct reflections is by adding potential barriers between the *symmetrically positioned waveguides* and the cavity. If the two barriers are equal, the system is fully LR symmetric; if the barriers are different, we have a LR-symmetric cavity coupled *asymmetrically* to the outside: using the jargon of nuclear physicists [7], we shall refer to this type of symmetry breaking as “external mixing,” with an obvious meaning. An interesting question, amenable to experimental observation, is that of the interplay between the symmetry of the cavity and external mixing in the statistical distribution of the conductance of such a structure: the study of that interplay is the second main purpose of this paper. From an experimental point of view, microwave cavities [8] and acoustic systems [9] might represent good candidates by which to study these questions.

That interplay may also be there and have interesting effects when  $\langle S \rangle = 0$ , as in the case of a LR-symmetric cavity coupled to the outside by two waveguides free of potential barriers but *asymmetrically located*. This problem can be addressed from the point of view of the systems described in the preceding paragraph in the following way. One may think of a LR-symmetric cavity coupled to the outside by four waveguides, also placed symmetrically. We can break the symmetry by providing the two waveguides on the right-hand side of the cavity, say, with identical barriers. The desired problem is then approached in the limit of impenetrable barriers.

This paper is organized as follows. In order to make the

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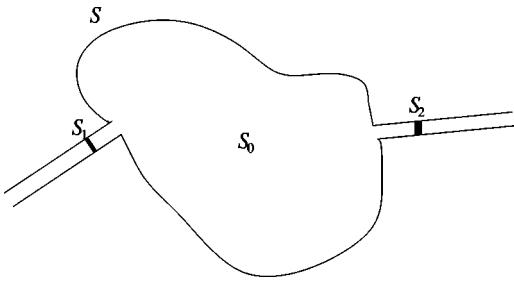


FIG. 1. A ballistic chaotic cavity with scattering matrix given by  $S_0$  connected to two waveguides by means of two barriers with scattering matrices  $S_1$  and  $S_2$ .

paper reasonably self-contained, we summarize in the next section a number of concepts that we shall be using throughout the paper, such as the invariant measure and Poisson's kernel for  $S$  matrices and their application to chaotic scattering in AS cavities, and the invariant measure for LR-symmetric systems. Section III deals with the problem of fully LR-symmetric systems in the presence of direct processes. The distribution of the conductance is calculated for the particular case of one open channel in each lead and a diagonal optical matrix (implying direct reflections), and contrasted with the one obtained for an AS chaotic cavity and the same optical matrix  $\langle S \rangle$ . Different barriers added to the two waveguides of an otherwise fully LR-symmetric system with no direct processes give rise to direct reflections and external mixing: the problem is studied in Sec. IV. Again, the conductance distribution is computed for the one-channel case and contrasted with the one obtained for an AS chaotic cavity with the same optical matrix  $\langle S \rangle$ . The problem of external mixing in a LR-symmetric cavity with asymmetrically positioned leads and  $\langle S \rangle = 0$  is addressed in Sec. V. The conductance distribution is calculated and compared with the one arising from the invariant measure in the AS case. Finally, for the sake of completeness, we include a number of appendices where some of the results mentioned in the text are derived.

## II. THE $S$ MATRIX AND ITS STATISTICAL DISTRIBUTION

### A. The scattering problem in the absence of spatial symmetries

A single-electron scattering problem can be described by the scattering matrix  $S$ , which in the stationary case relates the outgoing-wave to the incoming-wave amplitudes[10]. For a ballistic cavity connected to two leads, each with  $N$  transverse propagating modes (see Fig. 1), the  $S$  matrix is  $n=2N$  dimensional and has the structure

$$S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}, \quad (2.1)$$

where  $r, r'$  are the  $N \times N$  reflection matrices (for incidence from either lead) and  $t, t'$  the corresponding transmission matrices.

From the  $S$  matrix we can construct the total transmission coefficient, or *spinless dimensionless conductance*

$$T = \text{tr}(tt^\dagger), \quad (2.2)$$

which is proportional to the conductance of the cavity,

$$G = (2e^2/h)T, \quad (2.3)$$

the factor 2 arising from the two spin directions.

In Dyson's scheme [11] there are three basic symmetry classes. In the absence of any symmetry, the only restriction on  $S$  is unitarity, i.e.,

$$SS^\dagger = I, \quad (2.4)$$

resulting from the physical requirement of flux conservation. This is the "unitary" case, also designated as  $\beta=2$ . For orthogonal symmetry, or  $\beta=1$ ,  $S$  is symmetric, i.e.,

$$S = S^T, \quad (2.5)$$

because one has either time-reversal invariance (TRI) and integral spin, or TRI, half-integral spin and rotational symmetry. In the "symplectic" case ( $\beta=4$ ),  $S$  is self-dual because of TRI with half-integral spin and no rotational symmetry. From now on we consider the scattering problem of "spinless" electrons, so that the case  $\beta=4$  will not be touched upon.

A convenient parametrization of the  $S$  matrix is the polar representation [12,13]

$$S = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix} \begin{pmatrix} -\sqrt{1-\tau} & \sqrt{\tau} \\ \sqrt{\tau} & \sqrt{1-\tau} \end{pmatrix} \begin{pmatrix} v_3 & 0 \\ 0 & v_4 \end{pmatrix}, \quad (2.6)$$

where  $\tau$  stands for the  $N$ -dimensional diagonal matrix of eigenvalues  $\tau_a$  ( $a=1, \dots, N$ ) of the Hermitian matrix  $tt^\dagger$ ;  $v_i$  ( $i=1, \dots, 4$ ) are arbitrary  $N \times N$  unitary matrices for  $\beta=2$ , with the restriction  $v_3 = v_1^T$ ,  $v_4 = v_2^T$  for  $\beta=1$ .

### 1. The invariant measure

When the classical dynamics of the system is chaotic, a statistical analysis of the quantum-mechanical problem is called for. That analysis is performed in terms of "ensembles" of physical systems, described mathematically by an ensemble of  $S$  matrices, endowed with a probability measure. The starting point of such an analysis is the concept of *invariant measure*, which is a precise formulation of the intuitive notion of *equal a priori probabilities* in the space of scattering matrices.

The invariant measure, to be designated as  $d\mu^{(\beta)}(S)$ , is invariant under the symmetry operation relevant to the universality class under consideration [11,14], i.e.,

$$d\mu^{(\beta)}(S) = d\mu^{(\beta)}(U_0 S V_0). \quad (2.7)$$

Here,  $U_0, V_0$  are arbitrary but fixed unitary matrices in the unitary case, while  $V_0 = U_0^T$  in the orthogonal one. Equation (2.7) defines the circular (orthogonal, unitary) ensembles (COE, CUE), for  $\beta=1, 2$ , respectively.

## 2. Chaotic scattering by AS cavities

The information-theoretic approach of Refs. [15,16] leads to the probability distribution known as Poisson's kernel [6,14]:

$$dP_{\langle S \rangle}^{(\beta)}(S) = \frac{[\det(I - \langle S \rangle \langle S \rangle^\dagger)]^{(\beta n + 2 - \beta)/2}}{|\det(I - S \langle S \rangle^\dagger)|^{\beta n + 2 - \beta}} d\mu^{(\beta)}(S), \quad (2.8)$$

where the invariant measure is assumed normalized, i.e.,

$$\int d\mu^{(\beta)}(S) = 1. \quad (2.9)$$

Here,  $n = 2N$  is the dimensionality of the  $S$  matrix and  $\langle S \rangle$  is the averaged, or *optical*,  $S$  matrix, which describes the prompt response arising from *direct processes*.

In the absence of direct processes,  $\langle S \rangle = 0$  and Poisson's measure [Eq. (2.8)] reduces to the invariant measure for the universality class in question. In terms of the polar representation, the invariant measure can be written as [17,18]

$$d\mu^{(\beta)}(S) = p^{(\beta)}(\{\tau\}) \prod_a d\tau_a \prod_i d\mu(v_i). \quad (2.10)$$

Here, the joint probability density of  $\{\tau\}$  is

$$p^{(\beta)}(\{\tau\}) = C_\beta \prod_{a < b} |\tau_a - \tau_b|^\beta \prod_c \tau_c^{(\beta-2)/2}, \quad (2.11)$$

$C_\beta$  being a normalization constant and  $d\mu(v_i)$  denoting the invariant measure on the unitary group  $U(N)$  for matrices  $v_i$ .

For  $\langle S \rangle \neq 0$ , a useful construction of Poisson's ensemble is given in Refs. [19,20]. Consider the system shown in Fig. 1: it consists of a cavity described by the  $n$ -dimensional scattering matrix  $S_0$ , connected to two leads by the tunnel barriers described by the  $n \times n$  scattering matrices

$$S_1 = \begin{pmatrix} r_1 & t'_1 \\ t_1 & r'_1 \end{pmatrix}, \quad (2.12)$$

$$S_2 = \begin{pmatrix} r_2 & t'_2 \\ t_2 & r'_2 \end{pmatrix}, \quad (2.13)$$

respectively. We bunch the two leads into a "superlead" and construct the  $2n \times 2n$  scattering matrix  $S_b$

$$S_b = \begin{pmatrix} r_b & t'_b \\ t_b & r'_b \end{pmatrix} = \begin{pmatrix} r_1 & 0 & t'_1 & 0 \\ 0 & r'_2 & 0 & t_2 \\ t_1 & 0 & r'_1 & 0 \\ 0 & t'_2 & 0 & r_2 \end{pmatrix}. \quad (2.14)$$

Here, the various blocks ( $r_b$ , etc.) are  $n$  dimensional. The scattering matrix  $S_0$  for the cavity can be written in terms of the scattering matrix  $S$  for the full system {cavity + barriers} as

$$S_0 = \frac{1}{t'_b} (S - r_b) \frac{1}{I - r_b^\dagger S} t_b^\dagger. \quad (2.15)$$

One can prove [14,16,19,20] that between the invariant measures for  $S_0$  and for  $S$  we have the Jacobian

$$d\mu^{(\beta)}(S_0) = \frac{[\det(I - \langle S \rangle \langle S \rangle^\dagger)]^{(\beta n + 2 - \beta)/2}}{|\det(I - S \langle S \rangle^\dagger)|^{\beta n + 2 - \beta}} d\mu^{(\beta)}(S). \quad (2.16)$$

Now, if the matrix  $S_0$  for the cavity is distributed according to the invariant measure, i.e.,  $d\mu^{(\beta)}(S_0)$ , the distribution of the transformed  $S$  satisfies

$$dP(S) = d\mu^{(\beta)}(S_0) \quad (2.17)$$

and we obtain Eq. (2.8), the optical  $S$  being given by the  $n$ -dimensional matrix

$$\langle S \rangle = r_b = \begin{pmatrix} r_1 & 0 \\ 0 & r'_2 \end{pmatrix}. \quad (2.18)$$

*The  $N=1$ ,  $\beta=1$  case. The T distribution.* We now consider the distribution of the  $S$  matrix for the system shown in Fig. 1 for the case  $N=1$  and  $\beta=1$ . The matrices  $S_0$  of the ballistic cavity,  $S_1$  and  $S_2$  of the two tunnel barriers, and  $S$  [related through Eq. (2.15)] are  $2 \times 2$  and have the structure (2.1) with  $t' = t$ . In the polar representation (2.6) we have three independent parameters  $\tau$ ,  $\phi$ ,  $\psi$ , where we have written  $v_1 = e^{i\phi}$ ,  $v_2 = e^{i\psi}$ . The range of variation of these parameters is taken to be

$$\begin{aligned} \tau &\in [0,1], \\ \phi, \psi &\in [0, 2\pi]. \end{aligned} \quad (2.19)$$

In terms of (2.19),  $S$  can be written as

$$S = \begin{pmatrix} r & t \\ t & r' \end{pmatrix} = \begin{pmatrix} -\sqrt{1-\tau} e^{2i\phi} & \sqrt{\tau} e^{i(\phi+\psi)} \\ \sqrt{\tau} e^{i(\phi+\psi)} & \sqrt{1-\tau} e^{2i\psi} \end{pmatrix}, \quad (2.20)$$

and the invariant measure of Eqs. (2.10) and (2.11) as

$$d\mu^{(1)}(S) = \frac{d\tau}{2\sqrt{\tau}} \frac{d\phi}{2\pi} \frac{d\psi}{2\pi}. \quad (2.21)$$

The distribution of  $S$  is given by Poisson's kernel, with the optical  $S$  matrix

$$\langle S \rangle = r_b = \begin{pmatrix} r_1 & 0 \\ 0 & r'_2 \end{pmatrix}. \quad (2.22)$$

Substituting  $\langle S \rangle$  in Eq. (2.8), Poisson's measure can be written as

$$dP_{r_1, r_2}(S) = \frac{[(1 - |r_1|^2)(1 - |r_2'|^2)]^{3/2}}{|(1 - rr_1^*)(1 - r'r_2'^*) - t^2 r_1^* r_2'^*|^3} d\mu^{(1)}(S). \quad (2.23)$$

By definition, the resulting distribution of the transmission coefficient  $T$  can be expressed as the integral

$$w_{r_1, r'_2}(T) = \int \delta(T - \tau) dP_{r_1, r'_2}(S). \quad (2.24)$$

For this distribution, Ref. [6] gives the expression

$$w_{r_1, r'_2}(T) = \frac{1}{2\sqrt{T}} [(1 - |r_1|^2)(1 - |r'_2|^2)]^{3/2} \left\langle \frac{1}{|(e^{-i\varphi} + |r_1|\sqrt{1-T})(e^{-i\psi} + |r'_2|\sqrt{1-T}) - |r_1||r'_2|T|^3|} \right\rangle_{\varphi, \psi}, \quad (2.25)$$

where  $\langle \dots \rangle_{\varphi, \psi}$  denotes an average over the variables  $\varphi$  and  $\psi$  over the interval  $[0, 2\pi]$ . When  $r_1 = r'_2 = 0$ , the above expression (2.25) reduces to

$$w_{0,0}(T) = \frac{1}{2\sqrt{T}}, \quad (2.26)$$

as it should. Figure 2 below (Sec. III) shows with dotted lines the evolution of  $w_{r_1, r'_2}(T)$  for  $r_1 = r'_2 = \langle r \rangle$  with the parameter  $\langle r \rangle$ , obtained from Eq. (2.25) by numerical integration. That distribution tends to  $\delta(T)$  as  $\langle r \rangle \rightarrow -1$ .

To further illustrate the physics resulting from the  $S$ -matrix distribution (2.23) we analyze the special case  $r_1 = 0$ , so that the right barrier is the only one present. For this case, Eqs. (2.23) and (2.21) give, for the joint probability distribution of the parameters  $\tau, \phi, \psi$ , the expression

$$dP_{0, r'_2}(S) = \frac{(1 - |r'_2|^2)^{3/2}}{|1 - \sqrt{1 - \tau} e^{2i\psi} r'_2|^3} \frac{d\tau}{2\sqrt{\tau}} \frac{d\phi}{2\pi} \frac{d\psi}{2\pi}. \quad (2.27)$$

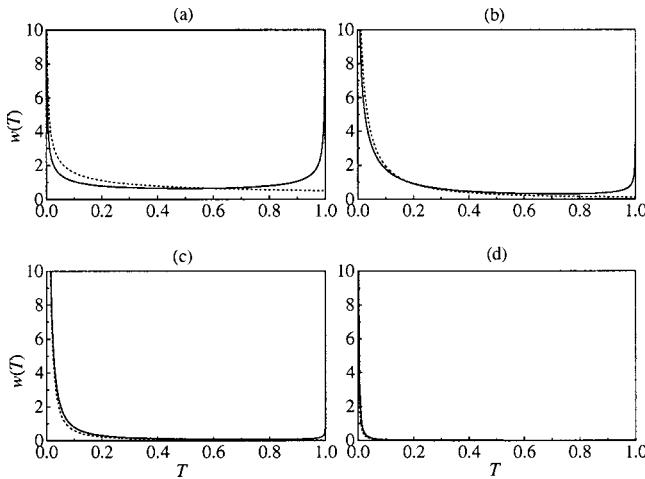


FIG. 2. Shown with a heavy line is the evolution of the distribution  $w_{\langle r \rangle}(T)$  of Eq. (3.11) with the parameter  $\langle r \rangle = -\cos \epsilon$  for a chaotic cavity with full LR symmetry. Cases  $\epsilon = \pi/2, \pi/4, \pi/8, \pi/32$  are shown in (a), (b), (c), (d), respectively. The dotted lines show for comparison the  $T$  distribution corresponding to an AS cavity with two identical barriers.

We first notice that the angular variable  $\phi$  is *uniformly distributed* for all  $r'_2$ . In this particular case the  $T$  probability density of Eq. (2.25) can be integrated analytically, to give [6]

$$w_{0, r'_2}(T) = \frac{(1 - |r'_2|^2)^{3/2}}{2\sqrt{T}} {}_2F_1[3/2; 3/2; 1; |r'_2|^2(1 - T)], \quad (2.28)$$

${}_2F_1$  being a hypergeometric function [21].

As a check, we consider two limiting situations. First, for  $r'_2 = 0$  we have a ballistic cavity without prompt response. The probability distribution for  $S$ ,  $dP_{0,0}(S)$  [see Eq. (2.27)], goes back to the invariant measure (2.21), as it should. Second, we obstruct the right lead by making the barrier there a perfect reflector. As a result,  $r'_2 = -1$  and it can be shown (see Appendix A) that  $dP_{0, r'_2}(S)$  reduces to

$$dP_{0, -1}(S) = \delta(\tau) d\tau \frac{d\phi}{2\pi} \frac{1}{2} \left[ \delta\left(\psi - \frac{\pi}{2}\right) + \delta\left(\psi - 3\frac{\pi}{2}\right) \right] d\psi, \quad (2.29)$$

where the angles in the arguments of the delta functions are defined modulo  $2\pi$ . We see from the above expression that the distribution of  $\tau$  is a one-sided delta function at zero, i.e.,

$$w(T) = \delta(T), \quad (2.30)$$

so that the transmission tends to zero, as expected. Also, the distribution of  $\psi$  consists of delta functions centered at  $\pi/2$  and  $3\pi/2$ , so as to ensure the vanishing of the wave function at the impenetrable barrier. In contrast, as already noted, the variable  $\phi$  is uniformly distributed from 0 to  $2\pi$ . In this limiting case we end up with a ballistic cavity connected to just one lead: thus *the resulting one-dimensional S matrix  $r = -e^{2i\phi}$  is distributed according to the invariant measure*.

Now we go back to the intermediate case in which  $r'_2$  in Eq. (2.28) is real and  $-1 < r'_2 < 0$ . We show in Fig. 5 below (Sec. IV) with dotted lines the evolution of the  $T$  distribution for several values of  $r'_2$ , obtained from the analytical result (2.28).

## B. The scattering problem for TRI, LR-symmetric systems

In the presence of additional symmetries, for fixed values for all quantum numbers of the full symmetry group the invariant ensemble is one of the three circular ensembles in

Dyson's scheme. Thus for reflection-symmetric systems  $S$  is block diagonal in a basis of definite parity with respect to reflections, with a circular ensemble in each block [4,5].

For a system with TRI and LR symmetry the general form of the  $S$  matrix is

$$S = \begin{pmatrix} r & t \\ t & r \end{pmatrix}, \quad (2.31)$$

with

$$r = r^T \quad (2.32a)$$

$$t = t^T. \quad (2.32b)$$

All the matrices with the structure (2.31) can be simultaneously brought to block-diagonal form using the rotation matrix

$$R_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} I_N & I_N \\ -I_N & I_N \end{pmatrix}, \quad (2.33)$$

where  $I_N$  is the  $N$ -dimensional unit matrix. In fact,

$$S' = R_0 S R_0^T = \begin{bmatrix} s^{(+)} & 0 \\ 0 & s^{(-)} \end{bmatrix}, \quad (2.34)$$

with

$$s^{(\pm)} = r \pm t. \quad (2.35)$$

Since  $S$  is unitary and symmetric, so are  $S'$  and the two  $N \times N$  matrices  $s^{(\pm)}$ . While  $S$  has the restricted form (2.31),  $s^{(\pm)}$  are the *most general*  $N \times N$  *unitary and symmetric*, i.e.,  $\beta=1$ , matrices.

### 1. The invariant measure

The invariant measure for  $S$  matrices with the structure (2.31) was found in Refs. [4,5], based on the consideration that two arbitrary unitary symmetric matrices  $s^{(\pm)}$  can generate the most general unitary  $S$  matrix with the structure (2.31). The invariant measure for matrices of the form (2.31) can be written as

$$d\hat{\mu}^{(1)}(S) = d\mu^{(1)}(s^{(+)}) d\mu^{(1)}(s^{(-)}), \quad (2.36)$$

where  $d\mu^{(1)}(s^{(\pm)})$  is the invariant measure discussed above for unitary and symmetric matrices ( $\beta=1$ ) in the absence of spatial symmetries.

### 2. Chaotic scattering by systems with full LR symmetry in the absence of direct processes

It has been found [5] that single-electron scattering by classically chaotic cavities with LR symmetry and in the absence of direct processes is well described by the invariant measure discussed above.

*The  $N=1$  case. The  $T$  distribution.* Reference [4] finds the distribution of the total transmission coefficient  $T$  for the

one-channel case ( $N=1$ ) arising from the invariant measure (2.36) as

$$w(T) = \frac{1}{\pi \sqrt{T(1-T)}}. \quad (2.37)$$

### III. SYSTEMS WITH TRI AND FULL LR SYMMETRY IN THE PRESENCE OF DIRECT PROCESSES

In this section we study a TRI system with full LR symmetry, just as in Sec. II B, but now admitting the possibility of direct processes. For the systems analyzed in Sec. II B, the average (or optical)  $S$  matrix  $\langle S \rangle$  vanishes, indicating the absence of a prompt response, whereas now  $\langle S \rangle \neq 0$ .

The  $S$  matrix has the structure of Eq. (2.31), and so does  $\langle S \rangle$ , i.e.,

$$\langle S \rangle = \begin{pmatrix} \langle r \rangle & \langle t \rangle \\ \langle t \rangle & \langle r \rangle \end{pmatrix}, \quad (3.1)$$

with

$$\langle r \rangle = \langle r \rangle^T, \quad (3.2a)$$

$$\langle t \rangle = \langle t \rangle^T \quad (3.2b)$$

being  $N \times N$  blocks. Both  $S$  and  $\langle S \rangle$  can be brought to a block-diagonal form by the rotation matrix (2.33):  $S$  becomes  $S'$  of Eq. (2.34) and  $\langle S \rangle$  becomes

$$\langle S' \rangle = R_0 \langle S \rangle R_0^T = \begin{bmatrix} \langle s^{(+)} \rangle & 0 \\ 0 & \langle s^{(-)} \rangle \end{bmatrix}. \quad (3.3)$$

As we noticed right below Eq. (2.35),  $s^{(\pm)}$  are the *most general*  $N \times N$  *unitary and symmetric* matrices; they thus belong to the  $\beta=1$  universality class. Their distribution is given by two statistically independent Poisson kernels of the form (2.8), with  $\langle s^{(\pm)} \rangle$  as their optical matrices. Denoting by  $d\hat{P}_{\langle S \rangle}(S)$  the  $S$  matrix distribution, we have

$$d\hat{P}_{\langle S \rangle}(S) = dP_{\langle s^{(+)} \rangle}(s^{(+)}) dP_{\langle s^{(-)} \rangle}(s^{(-)}), \quad (3.4)$$

where

$$\begin{aligned} dP_{\langle s^{(\pm)} \rangle}(s^{(\pm)}) \\ = \frac{[\det(I_N - \langle s^{(\pm)} \rangle \langle s^{(\pm)} \rangle^\dagger)]^{(N+1)/2}}{|\det(I_N - s^{(\pm)} \langle s^{(\pm)} \rangle^\dagger)|^{N+1}} d\mu^{(1)}(s^{(\pm)}). \end{aligned} \quad (3.5)$$

We can thus write  $d\hat{P}_{\langle S \rangle}(S)$  as

$$d\hat{P}_{\langle S \rangle}(S) = \frac{[\det(I_N - \langle s^{(+)} \rangle \langle s^{(+)} \rangle^\dagger)]^{(N+1)/2}}{|\det(I_N - s^{(+)} \langle s^{(+)} \rangle^\dagger)|^{N+1}} \frac{[\det(I_N - \langle s^{(-)} \rangle \langle s^{(-)} \rangle^\dagger)]^{(N+1)/2}}{|\det(I_N - s^{(-)} \langle s^{(-)} \rangle^\dagger)|^{N+1}} d\hat{\mu}^{(1)}(S) \quad (3.6)$$

where  $d\hat{\mu}^{(1)}(S)$  is defined in Eq. (2.36).

The special case of no direct transmission,  $\langle t \rangle = 0$ , i.e.,

$$\langle S \rangle = \begin{pmatrix} \langle r \rangle & 0 \\ 0 & \langle r \rangle \end{pmatrix}, \quad (3.7)$$

can be written as

$$d\hat{P}_{\langle r \rangle}(S) = \frac{[\det(I_N - \langle r \rangle \langle r \rangle^\dagger)]^{N+1}}{|\det(I_N - s^{(+)} \langle r \rangle^\dagger)|^{N+1} |\det(I_N - s^{(-)} \langle r \rangle^\dagger)|^{N+1}} \times d\hat{\mu}^{(1)}(S). \quad (3.8)$$

Physically, this case could be realized by fully LR-symmetric structures with no direct processes, to which identical barriers (with the  $\beta=1$  symmetry) are added in the two leads, each with a reflection matrix [see Eq. (2.18)]

$$r_1 = r'_2 = \langle r \rangle. \quad (3.9)$$

The situation is illustrated in Fig. 3 ahead but with equal barriers.

#### The $N=1$ case. The $T$ distribution.

In this case, Eq. (3.8) reduces to

$$d\hat{P}_{\langle r \rangle}(S) = \frac{[1 - \langle r \rangle \langle r \rangle^*]^2}{|1 - s^{(+)} \langle r \rangle^*|^2 |1 - s^{(-)} \langle r \rangle^*|^2} d\hat{\mu}^{(1)}(S), \quad (3.10)$$

where  $\langle r \rangle$  and  $s^{(\pm)}$  are now  $1 \times 1$  matrices, i.e., just complex numbers. The distribution of  $T$  can be obtained from the general expression (2.24). For  $\langle r \rangle$  real, some of the relevant steps are found in Appendix B, the final result being

$$w_{\langle r \rangle}(T) = \frac{1}{\pi \sqrt{T(1-T)}} \frac{(1 + \langle r \rangle^2)(1 - \langle r \rangle^2)}{(1 + \langle r \rangle^2)^2 - 4\langle r \rangle^2(1-T)}. \quad (3.11)$$

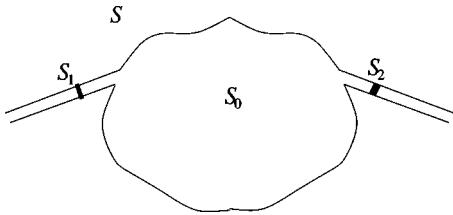


FIG. 3. A ballistic cavity with reflection symmetry described by the matrix  $S_0$ , connected to two waveguides by means of two barriers described by  $S_1, S_2$ . The barriers give rise to direct processes and, if they are different, the LR symmetry of the full system is broken (external mixing).

The distribution (3.11) is plotted in Fig. 2 for several values of  $\langle r \rangle$  and compared, in the same figure, with the distribution corresponding to an AS cavity with the same  $\langle S \rangle$ , as given by Eq. (2.25).

For  $\langle r \rangle = 0$ , the distribution of Eq. (3.11) reduces to that of Eq. (2.37), which is symmetric with respect to  $T = \frac{1}{2}$ , so that  $T$  and  $R = 1 - T$  are identically distributed; this feature is lost when  $\langle r \rangle \neq 0$ , as small  $T$ 's become more probable. As  $\langle r \rangle \rightarrow -1$ , both distributions shown in the figure (i.e., for LR-symmetric and AS systems) tend to  $\delta(T)$ .

#### IV. BREAKING THE REFLECTION SYMMETRY OF CAVITIES BY DIRECT PROCESSES

In this section we study a TRI configuration consisting of a ballistic cavity with LR symmetry and scattering matrix  $S_0$ , connected to two symmetrically positioned waveguides by means of barriers described by  $S_1$  and  $S_2$ , respectively; in general, the barriers are allowed to be different. This arrangement introduces direct reflections and a *breakdown of the reflection symmetry* (see Fig. 3). As a result, while the scattering matrix  $S_0$  of the cavity plus the symmetrically positioned waveguides, but not including the barriers, has the restricted structure (2.31), (2.32), the scattering matrix  $S$  of the total system including the barriers has the more general form (2.1), (2.5). Now,  $S$  is generated from  $S_0$  through the inverse of the relation (2.15); thus, varying  $S_0$  across its manifold of independent parameters, but keeping the barriers fixed, generates a matrix  $S$  that varies over a manifold with the same dimensionality. In what follows we restrict ourselves to the one-channel case ( $N=1$ ) in each lead. The matrices  $S_0$  can be expressed in terms of *two* independent continuous parameters (plus a discrete parameter  $\sigma$ ), as in Eq. (4.9) below, while  $S$  has the more general form (2.20); thus there should be an algebraic relation connecting the three continuous parameters  $\tau, \phi, \psi$  appearing in the latter equation.

We want  $S_0$  to be distributed according to the invariant measure  $d\hat{\mu}^{(1)}(S_0)$ . In principle, the transformation between  $S_0$  and  $S$  (for fixed  $S_1$  and  $S_2$ ) defines uniquely the resulting statistical distribution of  $S$ , to be called  $d\hat{P}(S)$  [see Eq. (4.21)]; for that purpose one could find the Jacobian of the transformation relating  $S$  to  $S_0$ , both matrices being subject to the restrictions explained in the previous paragraph. In what follows, though, we find it convenient to compute  $d\hat{P}(S)$  proceeding along a simpler route, taking advantage of the Jacobian between *unrestricted*  $S$  matrices that we already know from Eq. (2.16). In fact, the measure  $d\hat{\mu}^{(1)}(S_0)$  can be first expressed as the measure  $d\mu^{(1)}(S_0)$  of *unrestricted*  $S_0$  matrices of the form of Eq. (4.5) below times the appropriate delta functions that provide the required restriction [see Eq. (4.6)] among the three parameters  $\tau_0, \phi_0, \psi_0$ . Next, Eq.

(2.16) expresses  $d\mu^{(1)}(S_0)$  in terms of  $d\mu^{(1)}(S)$ , the factor in front of  $d\mu^{(1)}(S)$  in Eq. (2.16) being the Jacobian of the transformation from unrestricted  $S_0$  to unrestricted  $S$  matrices. Finally, the identity (4.21) gives the required distribution  $d\hat{P}(S)$  for the  $S$  matrices. We proceed to implement this scheme in detail.

The relationship between the scattering matrix  $S_0$  for the cavity and the matrix  $S$  for the full system is given by Eq. (2.15), with

$$t_b = t'_b = \begin{pmatrix} t_1 & 0 \\ 0 & t_2 \end{pmatrix}, \quad (4.1)$$

$$r_b = \begin{pmatrix} r_1 & 0 \\ 0 & r_2 \end{pmatrix}, \quad r'_b = \begin{pmatrix} r'_1 & 0 \\ 0 & r_2 \end{pmatrix}. \quad (4.2)$$

Here all the matrices are two-dimensional, so that the various entries are just complex numbers. We thus have

$$S_0 = t_b^{-1} (S - r_b) \frac{1}{I_2 - r_b^\dagger S} t_b^\dagger. \quad (4.3)$$

The matrix  $S$  has the structure (2.20), while  $S_0$  has the structure (2.31), i.e.,

$$S_0 = \begin{pmatrix} r_0 & t_0 \\ t_0 & r_0 \end{pmatrix}. \quad (4.4)$$

It will be useful to write  $S_0$  of Eq. (4.4) in the polar representation (2.20) as

$$S_0 = \begin{bmatrix} -\sqrt{1-\tau_0} e^{2i\phi_0} & \sqrt{\tau_0} e^{i(\phi_0 + \psi_0)} \\ \sqrt{\tau_0} e^{i(\phi_0 + \psi_0)} & \sqrt{1-\tau_0} e^{2i\psi_0} \end{bmatrix}. \quad (4.5)$$

However, the three parameters  $\tau_0$ ,  $\phi_0$ , and  $\psi_0$  are not independent. In fact, the structure of  $S_0$  given in Eq. (4.4) implies a relationship between the two angles  $\phi_0$  and  $\psi_0$ , i.e.,

$$e^{2i\psi_0} = -e^{2i\phi_0}, \quad (4.6)$$

or, taking the square root on both sides

$$e^{i\psi_0} = i\sigma e^{i\phi_0}, \quad (4.7)$$

where  $\sigma = \pm 1$ . Equivalently,

$$\psi_0 = \phi_0 + \sigma \frac{\pi}{2}, \quad \text{mod}(2\pi). \quad (4.8)$$

The most general form of  $S_0$  is thus

$$S_0 = - \begin{bmatrix} \sqrt{1-\tau_0} & i\sigma\sqrt{\tau_0} \\ i\sigma\sqrt{\tau_0} & \sqrt{1-\tau_0} \end{bmatrix} e^{2i\phi_0}, \quad (4.9)$$

written in terms of the independent parameters  $\tau_0$ ,  $\phi_0$  and the discrete variable  $\sigma$ , which have the range of variation

$$\tau_0 \in [0, 1], \quad (4.10a)$$

$$\phi_0 \in [0, 2\pi], \quad (4.10b)$$

$$\sigma = \pm 1. \quad (4.10c)$$

From Eqs. (2.33)–(2.35), the matrix  $S_0$  can be diagonalized by a  $\pi/4$  rotation to give

$$S'_0 = \begin{bmatrix} e^{i\theta_0^{(+)}} & 0 \\ 0 & e^{i\theta_0^{(-)}} \end{bmatrix}, \quad (4.11)$$

where

$$e^{i\theta_0^{(\pm)}} = r_0 \pm t_0 = -e^{2i\phi_0 \pm i\sigma\beta_0} \quad (4.12)$$

and

$$\beta_0 = \tan^{-1} \sqrt{\frac{\tau_0}{1-\tau_0}}, \quad -\frac{\pi}{2} \leq \beta_0 \leq \frac{\pi}{2}. \quad (4.13)$$

With the range of variation (4.10) for  $\tau_0$ ,  $\phi_0$ , and  $\sigma$ ,  $e^{i\theta_0^{(+)}}$  and  $e^{i\theta_0^{(-)}}$  cover twice the torus defined by the two angles  $\theta_0^{(+)}$ ,  $\theta_0^{(-)}$ .

Equation (4.12) is a transformation from the parameters  $\tau_0$ ,  $\phi_0$ , and  $\sigma$  to the parameters  $\theta_0^{(+)}, \theta_0^{(-)}$ , whose Jacobian can be written as

$$\frac{1}{2} \frac{d\theta_0^{(+)}}{2\pi} \frac{d\theta_0^{(-)}}{2\pi} = \frac{1}{2} \frac{d\tau_0}{\pi\sqrt{\tau_0(1-\tau_0)}} \frac{d\phi_0}{2\pi}. \quad (4.14)$$

Both sides of this last equation integrate to 1 if the left-hand side is integrated in the region  $\theta_0^{(+)}, \theta_0^{(-)} \in [0, 2\pi]$  and multiplied by 2 to account for the fact that the region is visited twice, and the right-hand side is integrated in the region specified by Eq. (4.10).

According to Eq. (2.36), the left-hand side of Eq. (4.14) represents the invariant measure for  $S_0$  matrices with LR symmetry. A function  $f(\tau_0, \phi_0, \sigma)$  can be translated into a function  $\tilde{f}(\theta_0^{(+)}, \theta_0^{(-)})$  using the transformation (4.12); its average over the  $S_0$  invariant measure can thus be written as

$$\begin{aligned} & \int_0^{2\pi} \frac{d\theta_0^{(+)}}{2\pi} \int_0^{2\pi} \frac{d\theta_0^{(-)}}{2\pi} \tilde{f}(\theta_0^{(+)}, \theta_0^{(-)}) \\ &= \frac{1}{2} \sum_{\sigma=\pm 1} \int_0^1 \frac{d\tau_0}{\pi\sqrt{\tau_0(1-\tau_0)}} \int_0^{2\pi} \frac{d\phi_0}{2\pi} f(\tau_0, \phi_0, \sigma). \end{aligned} \quad (4.15)$$

Here, on the left-hand side we integrate over the torus  $\theta_0^{(+)}, \theta_0^{(-)}$  only once. Suppose now that we are given a function  $F(\tau_0, \phi_0, \psi_0) = F'(\tau_0, \phi_0, e^{i\psi_0})$  of the three parameters appearing in Eq. (4.5) and we want to compute its average over the above measure. First, we make use of Eq. (4.7) to eliminate  $\psi_0$  and write

$$\begin{aligned} F(\tau_0, \phi_0, \psi_0) &= F'(\tau_0, \phi_0, e^{i\psi_0}) = F'(\tau_0, \phi_0, i\sigma e^{i\phi_0}) \\ &= f(\tau_0, \phi_0, \sigma) = \tilde{f}(\theta_0^{(+)}, \theta_0^{(-)}), \end{aligned} \quad (4.16)$$

where  $f(\tau_0, \phi_0, \sigma)$  and  $\tilde{f}(\theta_0^{(+)}, \theta_0^{(-)})$ , have the same meaning as in Eq. (4.15) above. The average of this function can thus be written as in (4.15) and subsequently as

$$\begin{aligned} & \frac{1}{2} \sum_{\sigma=\pm 1} \int_0^1 \frac{d\tau_0}{\pi\sqrt{\tau_0(1-\tau_0)}} \int_0^{2\pi} \frac{d\phi_0}{2\pi} F'(\tau_0, \phi_0, i\sigma e^{i\phi_0}) \\ &= \frac{1}{2} \int_0^1 \frac{d\tau_0}{\pi\sqrt{\tau_0(1-\tau_0)}} \int_0^{2\pi} \frac{d\phi_0}{2\pi} \int_0^{2\pi} d\psi_0 \left[ \delta\left(\psi_0 - \phi_0 - \frac{\pi}{2}\right) \right. \\ & \quad \left. + \delta\left(\psi_0 - \phi_0 - 3\frac{\pi}{2}\right) \right] F(\tau_0, \phi_0, \psi_0), \end{aligned} \quad (4.17)$$

where we have used Eq. (4.8). Comparing the left hand-side of Eq. (4.15) to the right-hand side of Eq. (4.17) we thus write

$$\begin{aligned} \frac{d\theta_0^{(+)}}{2\pi} \frac{d\theta_0^{(-)}}{2\pi} & \sim \frac{2 \left[ \delta\left(\psi_0 - \phi_0 - \frac{\pi}{2}\right) + \delta\left(\psi_0 - \phi_0 - 3\frac{\pi}{2}\right) \right]}{\sqrt{1-\tau_0}} \\ & \times \frac{d\tau_0}{2\sqrt{\tau_0}} \frac{d\phi_0}{2\pi} \frac{d\psi_0}{2\pi}, \end{aligned} \quad (4.18)$$

where the symbol  $\sim$  indicates that the two measures are equivalent when the left- and right-hand sides are used to integrate the functions  $\tilde{f}(\theta_0^{(+)}, \theta_0^{(-)})$  and  $F(\tau_0, \phi_0, \psi_0)$ , respectively, defined above. Obviously, the angles in the argument of the delta functions above are defined modulo  $2\pi$ . As we have already noticed, the left-hand side of Eq. (4.18) is the invariant measure  $d\hat{\mu}^{(1)}(S_0)$  for scattering matrices  $S_0$  of the form (4.11), i.e., for a LR-symmetric cavity. On the other hand, Eq. (2.21) shows that the last line of Eq. (4.18) is the

invariant measure  $d\mu^{(1)}(S_0)$  for scattering matrices  $S_0$  of the more general form (4.5). The relationship between the two measures is thus

$$\begin{aligned} d\hat{\mu}^{(1)}(S_0) & \sim \frac{2 \left[ \delta\left(\psi_0 - \phi_0 - \frac{\pi}{2}\right) + \delta\left(\psi_0 - \phi_0 - 3\frac{\pi}{2}\right) \right]}{\sqrt{1-\tau_0}} \\ & \times d\mu^{(1)}(S_0). \end{aligned} \quad (4.19)$$

Here, the delta functions restrict the space of unitary and symmetric matrices to the subspace of matrices of the form (4.9).

As was explained at the beginning of this section, we now express  $d\mu^{(1)}(S_0)$  in terms of  $d\mu^{(1)}(S)$  using Eq. (2.16). That equation reads, for the present case,

$$d\mu^{(1)}(S_0) = \frac{[\det(I_2 - r_b r_b^\dagger)]^{3/2}}{|\det(I_2 - S r_b^\dagger)|^3} d\mu^{(1)}(S). \quad (4.20)$$

We substitute this last equation into Eq. (4.19) and use Eq. (2.21) to express  $d\mu^{(1)}(S)$  in the polar representation. We also note that the measure  $d\hat{\mu}^{(1)}(S_0)$  appearing on the left-hand side of Eq. (4.19), i.e., the differential probability associated with the matrices  $S_0$  [having the form (4.4)] for the LR-symmetric cavity, must coincide with the differential probability  $d\hat{P}_{r_b}(S)$  we are looking for, associated with the transformed matrices  $S$  [having the form (2.20), but with the appropriate restrictions], i.e.,

$$d\hat{P}_{r_b}(S) = d\hat{\mu}^{(1)}(S_0). \quad (4.21)$$

We thus have

$$d\hat{P}_{r_b}(S) \sim 2 \frac{\delta\left(\psi_0 - \phi_0 - \frac{\pi}{2}\right) + \delta\left(\psi_0 - \phi_0 - 3\frac{\pi}{2}\right)}{\sqrt{1-\tau_0}} \frac{[\det(I_2 - r_b r_b^\dagger)]^{3/2}}{|\det(I_2 - S r_b^\dagger)|^3} \frac{d\tau}{2\sqrt{\tau}} \frac{d\phi}{2\pi} \frac{d\psi}{2\pi}. \quad (4.22)$$

It remains to express the variables  $\psi_0, \phi_0, \tau_0$  appearing in the delta-function arguments in terms of  $\psi, \phi, \tau$ . This is done in Appendix C for the particular case in which barrier 1 is transparent, so that its scattering matrix  $S_1$  of Eq. (2.12) is the Pauli matrix  $\sigma_x$ , and barrier 2 is described by Eq. (2.13) with *real* matrix elements. The result is

$$d\hat{P}_{0,r_2'}(S) \sim p_{r_2'}(\tau, \phi, \psi) d\tau d\phi d\psi, \quad (4.23)$$

with

$\alpha(\phi)$  being given by Eq. (C7). We recall that the angles in the arguments of the delta functions are defined modulo  $2\pi$ .

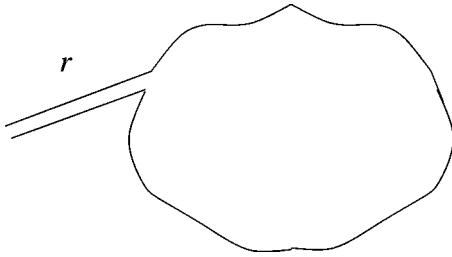


FIG. 4. A ballistic chaotic cavity with reflection symmetry connected to just one lead, supporting one open channel, in the absence of direct processes. The one-dimensional  $S$  matrix  $r = -e^{2i\phi}$  is distributed according to the invariant measure.

As a first check, set  $r'_2 = 0$ , corresponding to the case of no barriers. We obtain

$$d\hat{P}_{0,0}(S) \sim \frac{d\tau}{\pi\sqrt{\tau(1-\tau)}} \frac{d\phi}{2\pi} \frac{1}{2} \left[ \delta\left(\psi - \phi - \frac{\pi}{2}\right) + \delta\left(\psi - \phi - 3\frac{\pi}{2}\right) \right] d\psi. \quad (4.25)$$

Thanks to the delta functions, we recover the situation of LR symmetry. As expected, the right-hand side of Eq. (4.25) is the invariant measure defined for that symmetry, Eq. (4.18). As a second check, we analyze the case  $r'_2 \rightarrow -1$ , which corresponds to obstructing the waveguide on the right. We show in Appendix D that Eq. (4.23) gives in this case

$$d\hat{P}_{0,-1}(S) \sim \delta(\tau) d\tau \frac{d\phi}{2\pi} \frac{1}{2} \left[ \delta\left(\psi - \frac{\pi}{2}\right) + \delta\left(\psi - 3\frac{\pi}{2}\right) \right] d\psi. \quad (4.26)$$

The conductance distribution reduces to a one-sided delta function at zero, as it should. Notice that the variable  $\phi$  is uniformly distributed in the two extreme cases  $r'_2 = 0$  and  $r'_2 = -1$ ; this is not so for an arbitrary value of  $r'_2$ . In the limiting case  $r'_2 = -1$  we end up with a LR-symmetric ballistic cavity connected to just one lead (see Fig. 4): the resulting one-dimensional  $S$  matrix, i.e.,  $r = -e^{2i\phi}$ , is distributed according to the *invariant measure*: there is thus *no effect left of the LR symmetry of the cavity*. In fact, the right-hand side of Eq. (4.26) is identical to that of Sec. II A 2, Eq. (2.29), for an AS cavity with the right-hand waveguide obstructed. As we shall see later on, in Sec. V, this is a peculiarity of the one-channel case.

To get the joint distribution of  $\tau$  and  $\phi$  for arbitrary  $r'_2$  we integrate Eq. (4.24) over  $\psi$ . We find

$$q_{r'_2}(\tau, \phi) = \frac{1}{2\pi^2} \frac{(1-r'^2_2)^{3/2}}{\sqrt{\tau}} \frac{|\sqrt{1-\tau} - r'_2 e^{2i\phi}|}{|\sqrt{1-\tau}(1-r'^2_2) - r'_2 \tau e^{2i\phi}|^2}. \quad (4.27)$$

The  $T = \tau$  distribution  $w(T)$  is obtained by integrating  $q_{r'_2}(\tau, \phi)$  over  $\phi$ . Figure 5 shows (solid lines) the evolution of  $w(T)$  with the parameter  $r'_2$ . The peak in the solid curve in Fig. 5 seems to arise from the peak that is at  $T = 1$  origi-

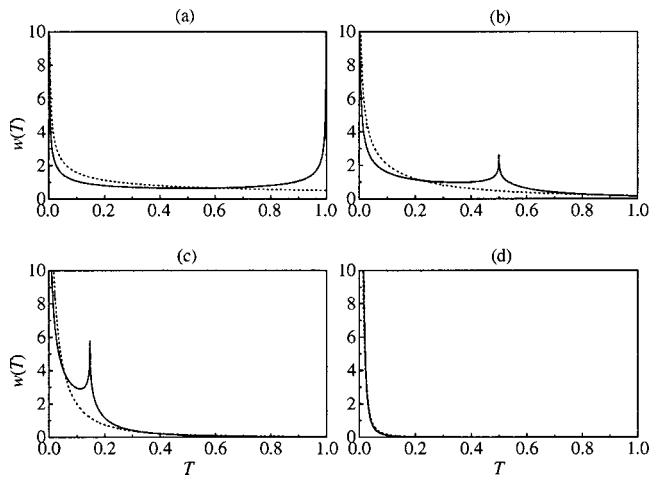


FIG. 5. The solid line shows the evolution of the  $T$  distribution obtained by numerical integration over  $\phi$  of Eq. (4.27) for a LR-symmetric cavity and one barrier defined by  $r'_2 = -\cos \epsilon$ , as a function of the parameter  $\epsilon$ . Panels (a), (b), (c), (d) show cases  $\epsilon = \pi/2, \pi/4, \pi/8, \pi/32$ , respectively. The dotted lines show, for comparison, the distributions that correspond to an AS cavity and the same barrier as for the corresponding heavy lines.

nally (i.e., for  $r'_2 = 0$ ); that peak then “travels” towards  $T = 0$  as  $|r'_2|$  increases. In the same figure we compare that sequence of distributions with those corresponding to an AS cavity (dotted lines) with the same  $r'_2$ . In the former case the system “remembers” in a rather conspicuous way that, although the resulting configuration is asymmetric, the cavity has LR symmetry.

## V. BREAKING THE REFLECTION SYMMETRY OF CAVITIES WITH AN ASYMMETRIC POSITION OF THE WAVEGUIDES

In the present section we study the effect of external mixing of LR symmetry in the absence of direct processes, i.e., for  $\langle S \rangle = 0$ : the problem will be that of a LR-symmetric cavity connected to two asymmetrically positioned waveguides in the absence of barriers. We proceed as follows. We first consider the LR-symmetric cavity connected to four symmetrically positioned waveguides (each supporting one open channel) by means of four, in general different, barriers, as shown in Fig. 6. The two barriers on the left-hand side are then removed, while those on the right are made perfect reflectors.

We call  $S_0$  the matrix associated with the LR-symmetric

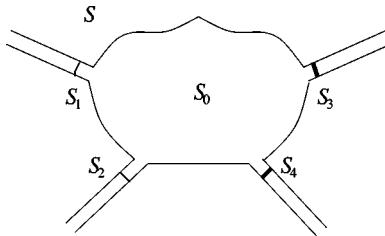


FIG. 6. A ballistic cavity connected to four waveguides symmetrically located, by means of four, in principle different, barriers.

cavity connected to the four symmetrically located waveguides in the *absence* of barriers. The matrix  $S$  in the presence of the four barriers is then given by Eq. (2.15), i.e.,

$$S = r_b + t'_b \frac{1}{I_4 - S_0 r'_b} S_0 t_b, \quad (5.1)$$

where

$$t_b = t'_b = \begin{pmatrix} t_1 & 0 & 0 & 0 \\ 0 & t_2 & 0 & 0 \\ 0 & 0 & t_3 & 0 \\ 0 & 0 & 0 & t_4 \end{pmatrix} \quad (5.2)$$

$$r_b = \begin{pmatrix} r_1 & 0 & 0 & 0 \\ 0 & r_2 & 0 & 0 \\ 0 & 0 & r'_3 & 0 \\ 0 & 0 & 0 & r'_4 \end{pmatrix}, \quad r'_b = \begin{pmatrix} r'_1 & 0 & 0 & 0 \\ 0 & r'_2 & 0 & 0 \\ 0 & 0 & r_3 & 0 \\ 0 & 0 & 0 & r_4 \end{pmatrix}. \quad (5.3)$$

As explained above, we now open the left waveguides and block the right ones by means of perfect reflectors, so that

$$t_b = t'_b = \begin{pmatrix} I_2 & 0_2 \\ 0_2 & 0_2 \end{pmatrix}, \quad r_b = r'_b = \begin{pmatrix} 0_2 & 0_2 \\ 0_2 & -I_2 \end{pmatrix}, \quad (5.4)$$

where  $I_2$  and  $0_2$  denote the two-dimensional unit and zero matrices, respectively.

The  $4 \times 4$  matrix  $S_0$  has the structure (2.31), i.e.,

$$S_0 = \begin{pmatrix} r_0 & t_0 \\ t_0 & r_0 \end{pmatrix}, \quad (5.5)$$

where  $r_0$  and  $t_0$  are two-dimensional matrices. The matrix  $S$  of Eq. (5.1) then reads

$$S = \begin{pmatrix} r_0 - t_0 \frac{1}{I_2 + r_0} t_0 & 0 \\ 0 & -I_2 \end{pmatrix}. \quad (5.6)$$

The 1-1 block of the above expression is the  $2 \times 2$  scattering matrix of the final system consisting of a LR-symmetric ballistic cavity connected to two waveguides on the left (see Fig. 7), i.e.,

$$s = r_0 - t_0 \frac{1}{I_2 + r_0} t_0. \quad (5.7)$$

Using the result (2.35) we can express  $r_0$  and  $t_0$  as

$$r_0 = \frac{1}{2} [s^{(+)} + s^{(-)}], \quad (5.8)$$

$$t_0 = \frac{1}{2} [s^{(+)} - s^{(-)}], \quad (5.9)$$

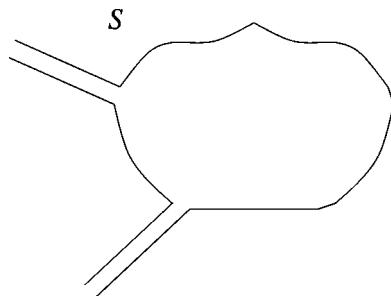


FIG. 7. A ballistic chaotic cavity connected to two asymmetrically located waveguides, without direct processes. The distribution of the two-dimensional  $S$  matrix is *not* the invariant measure, but is close to it.

where  $s^{(\pm)}$  are  $2 \times 2$  unitary and symmetric matrices.

A numerical calculation was performed, in which four-dimensional  $S_0$  matrices were generated with a distribution corresponding to their invariant measure: this was done by constructing an ensemble of  $s^{(\pm)}$  matrices, Eqs. (5.8), (5.9), distributed as two independent COE's. From Eq. (5.7), the resulting  $S$  matrices were then evaluated.

The distribution of the resulting transmission coefficient  $T$  is shown in Fig. 8. For comparison, the distribution  $1/2\sqrt{T}$  corresponding to an AS cavity connected to two one-channel waveguides and with  $\langle S \rangle = 0$  is shown with a dotted line. Although the LR-symmetric cavity with external symmetry breaking has a  $T$  distribution very close to  $1/2\sqrt{T}$ , there is a statistically significant deviation which indicates that the resulting system has a memory of the point symmetry of the cavity. This is to be contrasted with the result mentioned just before Eq. (4.27) for the single one-channel cavity illustrated in Fig. 4.

## VI. RESULTS AND CONCLUSIONS

One of the main purposes of the present paper has been the extension of previous studies on transport through ballistic chaotic cavities with reflection symmetry to include the presence of direct processes. In Sec. III we treated the problem of *fully left-right (LR)-symmetric systems* in the presence of direct processes and for the time-reversal invariant (TRI)

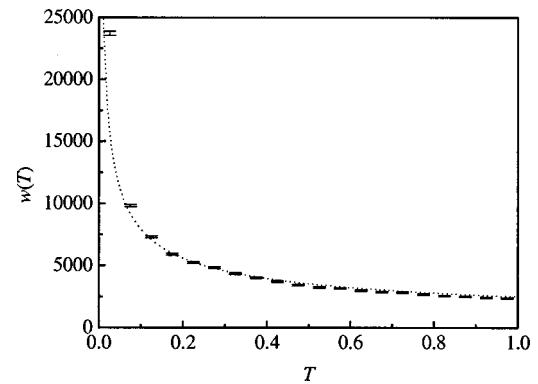


FIG. 8.  $T$  distribution for a LR-symmetric cavity connected to two asymmetrically located waveguides. The dotted line corresponds to an AS cavity connected to two waveguides.

case. The statistical distribution of the  $S$  matrix, found analytically in Eq. (3.6), consists of the product of two Poisson kernels with the optical matrices  $\langle s^{(+)} \rangle$  and  $\langle s^{(-)} \rangle$ , respectively. For no direct transmission processes,  $\langle t \rangle = 0$ , and real direct reflections  $\langle r \rangle$ , we calculated analytically the distribution of the transmission coefficient  $w(T)$  for the one-channel case. The difference with the  $T$  distribution for an asymmetric cavity (AC) with the same optical matrix  $\langle S \rangle$ , which is large for  $\langle r \rangle = 0$ , becomes less dramatic as  $|\langle r \rangle|$  increases; that evolution is shown in Fig. 2.

The case of a full LR-symmetric system with the TRI symmetry broken by a magnetic field is not addressed here. Reference [5] finds that the corresponding  $S$  matrix has a structure similar to that for the  $\beta=1$  case but with the roles of  $r$  and  $t$  interchanged. The statistical distribution  $w(T)$  of  $T$  for  $\langle r \rangle = 0$  is given by  $1/2\sqrt{1-T}$ , indicating coherent forward scattering. Starting from that distribution,  $w(T)$  would evolve towards  $\delta(T)$  as  $|\langle r \rangle|$  increases.

The other main purpose of this work has been the study of LR symmetry breaking by an asymmetric coupling of a LR-symmetric cavity to the outside. Two ways of producing external mixing of the spatial symmetry were analyzed:

(a) In Sec. IV we studied the effect of *breaking the reflection symmetry of a cavity by direct processes*. The system consists of a ballistic cavity with reflection symmetry connected to two symmetrically positioned waveguides by means of barriers which, in general, are allowed to be different (Fig. 3). We found analytically, in Eqs. (4.23) and (4.24), the statistical distribution of the  $S$  matrix for the one-channel case in each waveguide and, for simplicity, when only the barrier in the right-hand waveguide is present ( $r'_2 \neq 0$ ). The  $T$  distribution is very different from that for the fully AS case (i.e., the one in which the cavity itself is AS) having the same optical  $\langle S \rangle$  matrix, as shown in Fig. 5 for various values of  $r'_2 \neq 0$ . We conclude that this two-waveguide system, although asymmetric with respect to the LR operation, *has a memory of the reflection symmetry of the cavity* from which it is constructed. In the limit  $r'_2 \rightarrow -1$  the right-hand waveguide is blocked and we end up with a LR-symmetric ballistic cavity connected, without any barrier, to just one lead, supporting one open channel (see Fig. 4). We found that the resulting one-dimensional matrix  $S = e^{i\theta}$  is distributed according to its invariant measure (i.e.,  $\theta$  is uniformly distributed) and, as a result, *there is no effect left of the LR symmetry of the cavity*: this was found, though, to be a peculiarity of the one-waveguide–one-channel case (in fact, see the end of the next paragraph).

(b) In Sec. V we studied, *in the absence of direct processes, the effect of external mixing of LR symmetry induced by an asymmetric position of the waveguides*. The result is a LR-symmetric cavity connected, without any barriers, to two waveguides on its left-hand side (see Fig. 7). Let  $T$  denote the total transmission coefficient between those two waveguides; its distribution  $w(T)$  was calculated numerically for the one-channel case in each waveguide and compared, in Fig. 8, with  $1/2\sqrt{T}$ , the  $T$  distribution arising from the invariant measure  $d\mu^{(\beta=1)}(S)$  for AS systems. Although the difference between the two distributions is quite small, it

is statistically significant. This problem is clearly equivalent to having, on one side of the cavity, just one waveguide (coupled to the cavity without any barrier) supporting two open channels. In this one-waveguide–two-channel problem the resulting  $S$  matrix is thus distributed very closely to its invariant measure, the difference exhibiting *some memory left of the reflection symmetry of the cavity*.

Two additional points are worth mentioning. First, from an experimental point of view, we note that microwave cavities and acoustic systems might represent good systems with which to study the interplay between the symmetry of the cavity and external mixing in the statistical distribution of the conductance of such a structure. Finally, the problem described in b above is relevant to the study of transport between two one-channel leads connected by a “double” Cayley tree [22]. In fact, under suitable circumstances the two problems can be mapped unto each other. This problem will be reported on elsewhere.

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## APPENDIX A: DERIVATION OF EQ. (2.29)

We saw in Sec. II A 2 that the distribution  $dP_{\langle S \rangle}(S)$  of the scattering matrix of a cavity connected to two waveguides, where the one on the right of the cavity has a barrier, is given by

$$dP_{0,r'_2}(S) = \frac{(1-|r'_2|^2)^{3/2}}{|1-\sqrt{1-\tau e^{2i\psi}}r'_2|^3} \frac{d\tau}{2\sqrt{\tau}} \frac{d\phi}{2\pi} \frac{d\psi}{2\pi}. \quad (A1)$$

To see the behavior of  $dP_{0,r'_2}(S)$  for  $r'_2 = -1$ , let  $r'_2$  be a real number: assume for simplicity  $r'_2 = -\cos \epsilon$ ; we are interested in the limit  $\epsilon \rightarrow 0$ . Also, let us introduce the positive parameter  $\eta \ll 1$  in order to avoid the singularity at  $\tau = 0$ . Of course, we will take the limit  $\eta \rightarrow 0$  later on. Because the variable  $\phi$  is uniformly distributed, the joint probability density of  $\tau$  and  $\psi$  can be written as

$$p_{\eta,\epsilon}(\tau, \psi) = \frac{C_\eta}{4\pi\sqrt{\tau+\eta^2}} \frac{|\sin \epsilon|^3}{|1+\cos \epsilon\sqrt{1-\tau e^{2i\psi}}|^3}, \quad (A2)$$

where  $C_\eta$  is a normalization constant that depends on the parameter  $\eta$ .

We have the following properties of  $p_{\eta,\epsilon}(\tau, \psi)$ .

(1) From (A2) we see that

$$p_{0,0}(\tau, \psi) = \lim_{\eta \rightarrow 0} \lim_{\epsilon \rightarrow 0} p_{\eta,\epsilon}(\tau, \psi) = 0 \quad (A3)$$

for all  $\tau$  and  $\psi$ , except for  $\tau=0$  and  $\psi=\pi/2, 3\pi/2$ , where the denominator is zero:

$$|1+\sqrt{1-\tau e^{2i\psi}}|^3 = 0. \quad (A4)$$

(2) For  $\tau=0$  and  $\psi=\pi/2, 3\pi/2$  we have

$$\begin{aligned}
p_{0,0} \left( \tau=0, \psi=\frac{\pi}{2}, 3\frac{\pi}{2} \right) &= \lim_{\eta \rightarrow 0} \lim_{\epsilon \rightarrow 0} p_{\eta, \epsilon} \left( \tau=0, \psi=\frac{\pi}{2}, 3\frac{\pi}{2} \right) \\
&= \lim_{\eta \rightarrow 0} \lim_{\epsilon \rightarrow 0} \frac{C_\eta}{4\pi\eta} \left| \cot \frac{\epsilon}{2} \right|^3 \rightarrow \infty.
\end{aligned} \tag{A5}$$

(3) The function  $p_{0,0}(\tau, \psi)$  is normalized to unity:

$$\int_0^1 d\tau \int_0^{2\pi} d\psi p_{0,0}(\tau, \psi) = \lim_{\eta, \epsilon \rightarrow 0} \int_0^1 d\tau \int_0^{2\pi} d\psi p_{\eta, \epsilon}(\tau, \psi) = 1. \tag{A6}$$

Then the only function which satisfies those conditions is

$$p_{0,0}(\tau, \psi) = \delta(\tau) \frac{1}{2} \left[ \delta \left( \psi - \frac{\pi}{2} \right) + \delta \left( \psi - \frac{3\pi}{2} \right) \right]. \tag{A7}$$

Finally, the distribution of the  $S$  matrix in the above limits is given by Eq. (2.29).

## APPENDIX B: DERIVATION OF EQ. (3.11)

For  $\langle r \rangle$  real and  $s^{(\pm)} = e^{i\theta^{(\pm)}}$ , Eq. (3.10) can be written as

$$d\hat{P}_{\langle r \rangle}(S) = \frac{1 - \langle r \rangle^2}{|1 - \langle r \rangle e^{i\theta^{(+)}}|^2} \frac{1 - \langle r \rangle^2}{|1 - \langle r \rangle e^{i\theta^{(-)}}|^2} \frac{d\theta^{(+)}}{2\pi} \frac{d\theta^{(-)}}{2\pi}. \tag{B1}$$

The transmission amplitude is given by [see Eq. (2.35)]

$$t = \frac{1}{2} (e^{i\theta^{(+)}} - e^{i\theta^{(-)}}), \tag{B2}$$

and the transmission coefficient is written as

$$T = |t|^2 = \frac{1}{2} [1 - \cos(\theta^{(+)}) - \cos(\theta^{(-)})]. \tag{B3}$$

The  $T$  distribution  $w_{\langle r \rangle}(T)$  is obtained from

$$w_{\langle r \rangle}(T) = \int \delta \left\{ T - \frac{1}{2} [1 - \cos(\theta^{(+)}) - \cos(\theta^{(-)})] \right\} d\hat{P}_{\langle r \rangle}(S). \tag{B4}$$

In order to solve the integral, we make the change of variables

$$\theta = \frac{1}{2} [\theta^{(+)} - \theta^{(-)}], \tag{B5}$$

$$\theta' = \frac{1}{2} [\theta^{(+)} - \theta^{(-)}], \tag{B5}$$

the range of variation being: for  $\theta' \in (0, 2\pi)$ ,  $\theta \in (-\theta', \theta')$  and for  $\theta' \in (\pi, 2\pi)$ ,  $\theta \in (-2\pi + \theta', 2\pi - \theta')$ .

Substituting (B1) in (B4), considering the fact that the integrand is an even function of  $\theta$  and writing the delta function in terms of its roots in the variable  $\theta$ , we have

$$\delta(T - \sin^2 \theta) = \frac{1}{2\sqrt{T(1-T)}} [\delta(\theta - \theta_1) + \delta(\theta - \theta_2)], \tag{B6}$$

where  $\theta_2 = \pi - \theta_1$  and  $\theta_1 = \arcsin \sqrt{T}$ ; finally, after some algebra,  $w_{\langle r \rangle}(T)$  can be written as a sum of two terms:

$$w_{\langle r \rangle}(T) = \frac{(1 - \langle r \rangle^2)^2}{\pi^2 \sqrt{T(1-T)}} [I_1(T, \langle r \rangle) + I_2(T, \langle r \rangle)], \tag{B7}$$

where, for  $k = 1, 2$ ,

$$\begin{aligned}
I_k(T, \langle r \rangle) &= \int_0^\pi d\theta' \int_0^{\theta'} d\theta \frac{1}{[(1 + \langle r \rangle^2) - 2\langle r \rangle \cos(\theta' + \theta)]} \\
&\times \frac{\delta(\theta - \theta_k)}{[(1 + \langle r \rangle^2) - 2\langle r \rangle \cos(\theta' - \theta)]}.
\end{aligned} \tag{B8}$$

Again, after some algebra the sum of the two integrals give a single one:

$$I_1(T, \langle r \rangle) + I_2(T, \langle r \rangle) = \frac{1}{c} \int_0^\pi \frac{d\theta'}{a - b\cos \theta' + \cos^2 \theta'}, \tag{B9}$$

where

$$\begin{aligned}
a &= \frac{1}{c} [(1 + \langle r \rangle^2)^2 - 4\langle r \rangle^2 T], \\
b &= \frac{4}{c} \langle r \rangle (1 + \langle r \rangle^2) \sqrt{1 - T}, \\
c &= 4\langle r \rangle^2.
\end{aligned} \tag{B10}$$

Now, making the change of variable  $x = \cos \theta'$ , (B7) can be written as

$$w_{\langle r \rangle}(T) = \frac{(1 - \langle r \rangle^2)^2}{4\langle r \rangle^2 \pi^2 \sqrt{T(1-T)}} [I_+(T, \langle r \rangle) + I_-(T, \langle r \rangle)], \tag{B11}$$

where now

$$I_{\pm}(T, \langle r \rangle) = \int_0^1 \frac{dx}{\sqrt{1-x^2}(a \pm bx + x^2)}. \tag{B12}$$

By means of a change of variables

$$u = -\frac{x + (A + B)}{x + (A - B)}, \tag{B13a}$$

$$v = -\frac{x - (A + B)}{x - (A - B)}, \tag{B13b}$$

where

$$A = \frac{1}{b}(1+a), \quad (B14)$$

$$B = \frac{1}{b}\sqrt{(1+a)^2 - b^2}, \quad (B15)$$

the indefinite integrals,  $\text{Indef}_{\pm}$ , corresponding to each one of the above, can be transformed to

$$\text{Indef}_+ = \frac{2B}{\sqrt{CD}} \int \frac{|u+1|}{\sqrt{u^2+p(u^2+q)}} du, \quad (B16a)$$

$$\text{Indef}_- = -\frac{2B}{\sqrt{CD}} \int \frac{|v+1|}{\sqrt{v^2+p(v^2+q)}} dv, \quad (B16b)$$

where

$$p = \frac{a-b(B+A)+(B+A)^2}{a+b(B-A)+(B-A)^2} \quad (B17)$$

$$q = \frac{1-(B+A)^2}{1-(B-A)^2} \quad (B18)$$

and

$$C = 1 - (B-A)^2, \quad (B19)$$

Although the integrals (B12) seem to give the same result under the change  $b \rightarrow -b$ , they do not, because the cutoff  $x_u = B-A$  in (B16a), and  $x_v = A-B$  in (B16b), are different. One must be careful when evaluating the integrals in the limits. The results are

$$I_+(T, \langle r \rangle) = \frac{2B}{\sqrt{CD} \sqrt{p-q}} \left[ \arctan \left( \frac{1-\langle r \rangle^2}{2\langle r \rangle \sqrt{1-T}} \right) - \frac{1}{2\sqrt{p}} \ln \left( \frac{1+\langle r \rangle^2 + 2\langle r \rangle \sqrt{T}}{1+\langle r \rangle^2 - 2\langle r \rangle \sqrt{T}} \right) \right], \quad (B20a)$$

$$I_-(T, \langle r \rangle) = \frac{2B}{\sqrt{CD} \sqrt{p-q}} \left[ \pi - \arctan \left( \frac{1-\langle r \rangle^2}{2\langle r \rangle \sqrt{1-T}} \right) + \frac{1}{2\sqrt{p}} \ln \left( \frac{1+\langle r \rangle^2 + 2\langle r \rangle \sqrt{T}}{1+\langle r \rangle^2 - 2\langle r \rangle \sqrt{T}} \right) \right]. \quad (B20b)$$

Now, we substitute the sum of equations (B20) in (B11) to obtain the result

$$w_{\langle r \rangle}(T) = \frac{(1-\langle r \rangle^2)^2}{4\langle r \rangle^2 \pi^2 \sqrt{T(1-T)}} \frac{2\pi B}{\sqrt{CD} \sqrt{p-q}}; \quad (B21)$$

using Eqs. (B10), (B15), (B17), (B18), and (B19) the final result (3.11) is obtained.

### APPENDIX C: DERIVATION OF EQS. (4.23),(4.24)

For the particular case in which barrier 1 is transparent (see Fig. 3), so that its scattering matrix  $S_1$  of Eq. (2.12) is the Pauli matrix  $\sigma_x$ , and barrier 2 is described by Eq. (2.13) with *real* matrix elements, Eq. (4.22) can be written as

$$d\hat{P}_{0,r'_2}(S) \sim 2 \frac{\delta\left(\psi_0 - \phi_0 - \frac{\pi}{2}\right) + \delta\left(\psi_0 - \phi_0 - 3\frac{\pi}{2}\right)}{\sqrt{1-\tau_0}} \times \frac{(1-r'^2_2)^{3/2}}{|1-\sqrt{1-\tau}r'_2 e^{2i\psi}|^3} \frac{d\tau}{2\sqrt{\tau}} \frac{d\phi}{2\pi} \frac{d\psi}{2\pi}. \quad (C1)$$

Also, the transformation  $S_0(S)$  given by Eq. (4.3) can be written in terms of its elements as follows:

$$r_0 = \frac{1}{1-r'_2 r'} [r(1-r'_2 r') + r'_2 t^2],$$

$$r'_0 = \frac{1}{1-r'_2 r'} (r' - r'_2),$$

$$t_0 = \frac{1}{1-r'_2 r'} t_2 t, \quad (C2)$$

or in terms of the independent parameters [see Eqs. (2.20) and (4.5)] as

$$\sqrt{1-\tau_0} e^{2i\phi_0} = e^{2i\phi} \frac{\sqrt{1-\tau} - r'_2 e^{2i\psi}}{1-r'_2 \sqrt{1-\tau} e^{2i\psi}}, \quad (C3a)$$

$$\sqrt{1-\tau_0} e^{2i\psi_0} = e^{2i\psi} \frac{\sqrt{1-\tau} - r'_2 e^{-2i\psi}}{1-r'_2 \sqrt{1-\tau} e^{2i\psi}}, \quad (C3b)$$

$$\sqrt{\tau_0} e^{i(\phi_0 + \psi_0)} = \frac{t_2 \sqrt{\tau} e^{i(\phi + \psi)}}{1-r'_2 \sqrt{1-\tau} e^{2i\psi}}. \quad (C3c)$$

From (C3a) or (C3b) we find

$$\sqrt{1-\tau^{(0)}} = \frac{|\sqrt{1-\tau} - r'_2 e^{2i\psi}|}{|1 - \sqrt{1-\tau} r'_2 e^{2i\psi}|}; \quad (C4)$$

also, dividing (C3a) by (C3b) we obtain

$$e^{2i(\psi_0 - \phi_0)} = e^{2i(\psi - \phi)} \frac{\sqrt{1-\tau} - r'_2 e^{-2i\psi}}{\sqrt{1-\tau} - r'_2 e^{2i\psi}}. \quad (C5)$$

Because the roots of the delta functions appearing in Eq. (4.22) satisfy  $e^{2i(\psi_0 - \phi_0)} = -1$ , from (C5) we find

$$e^{2i\psi} = -e^{2i\phi} e^{2i\alpha(\phi)}, \quad (C6)$$

where

$$e^{i\alpha(\phi)} = \frac{\sqrt{1-\tau} - r'_2 e^{-2i\phi}}{|\sqrt{1-\tau} - r'_2 e^{2i\phi}|}. \quad (C7)$$

Then, we have the conditions for  $\psi$ :

$$\psi - \phi - \alpha(\phi) = \frac{\pi}{2} \quad \text{for} \quad \psi_0 - \phi_0 = \frac{\pi}{2}$$

$$\psi - \phi - \alpha(\phi) = 3\frac{\pi}{2} \quad \text{for} \quad \psi_0 - \phi_0 = 3\frac{\pi}{2}. \quad (C8)$$

The Jacobian for the transformation  $\psi_0 \rightarrow \psi$  is

$$\left| \frac{\partial}{\partial \psi} (\psi_0 - \phi_0) \right| = \frac{|(1-\tau) - r'_2|^2}{|\sqrt{1-\tau} - r'_2 e^{-2i\psi}|^2}. \quad (C9)$$

Then we write

$$\begin{aligned} \delta\left(\psi_0 - \phi_0 - \frac{2n+1}{2}\pi\right) &= \frac{|\sqrt{1-\tau} - r'_2 e^{-2i\psi}|^2}{|(1-\tau) - r'_2|^2} \\ &\times \delta\left[\psi - \phi - \alpha(\phi) - \frac{2n+1}{2}\pi\right], \end{aligned} \quad (C10)$$

for  $n=0,1$ .

From (C6) and (C7) we find

$$\sqrt{1-\tau} - r'_2 e^{2i\psi} = \frac{(1-\tau) - r'_2}{\sqrt{1-\tau} - r'_2 e^{2i\phi}}, \quad (C11)$$

$$1 - r'_2 \sqrt{1-\tau} e^{2i\psi} = \frac{\sqrt{1-\tau}(1 - r'_2) - r'_2 \tau e^{2i\phi}}{\sqrt{1-\tau} r'_2 e^{2i\phi}}. \quad (C12)$$

Finally, substituting Eqs. (C4), (C10), (C11), and (C12) in Eq. (C1), we arrive at

$$d\hat{P}_{0,r'_2}(\tau, \phi, \psi) \sim p_{r'_2}(\tau, \phi, \psi) d\tau d\phi d\psi, \quad (C13)$$

where  $p_{r'_2}(\tau, \phi, \psi)$  is given by Eq. (4.24).

#### APPENDIX D: DERIVATION OF EQ. (4.26)

In Sec. IV we find the joint distribution of  $\tau$ ,  $\phi$ , and  $\psi$  [Eq. (4.24)]. It is easy to integrate that distribution over  $\psi$  to find the joint distribution of  $\tau$  and  $\phi$  as

$$q_{r'_2}(\tau, \phi) = \frac{(1 - r'_2)^{3/2} |\sqrt{1-\tau} - r'_2 e^{2i\phi}|}{(2\pi)^2 \sqrt{\tau} |\sqrt{1-\tau}(1 - r'_2) - r'_2 \tau e^{2i\phi}|^2}. \quad (D1)$$

As in Appendix A, we assume for simplicity  $r'_2 = -\cos \epsilon$ ; again we introduce the parameter  $\eta \ll 1$ . Of course, we will take the limits  $\eta, \epsilon \rightarrow 0$ ; then

$$q_{\eta, \epsilon}(\tau, \phi) = \frac{C_\eta}{2\pi^2} \frac{|\sin \epsilon|^3}{\sqrt{\tau + \eta^2}} \frac{|\sqrt{1-\tau} + \cos \epsilon e^{2i\phi}|}{|\sqrt{1-\tau} \sin^2 \epsilon + \tau \cos \epsilon e^{2i\phi}|^2}. \quad (D2)$$

where  $C_\eta$  is a normalization constant that depends on  $\eta$ .

Again, as before, we have the following properties for  $q_{\eta, \epsilon}(\tau, \phi)$ :

(1) From (D2) we see that

$$q_{0,0}(\tau, \phi) = \lim_{\eta \rightarrow 0} \lim_{\epsilon \rightarrow 0} q_{\eta, \epsilon}(\tau, \phi) = 0 \quad (D3)$$

for all  $\tau$  and  $\phi$ , except for  $\tau=0$  and  $\phi=\pi/2, 3\pi/2$ , where the denominator is zero:

$$|\sqrt{1-\tau} \sin^2 \epsilon + \tau \cos \epsilon e^{2i\phi}|^2 = 0. \quad (D4)$$

(2)  $\tau \neq 0$  and  $\forall \phi$ .

It is easy to see from (D2) that in this case

$$q_{0,0}(\tau \neq 0, \phi) = \lim_{\eta} \lim_{\epsilon \rightarrow 0} q_{\eta, \epsilon}(\tau, \phi) = 0. \quad (D5)$$

(3) For  $\tau=0$  and  $\phi=\pi/2, 3\pi/2$  we have

$$\begin{aligned} q_{0,0}\left(\tau=0, \phi = \frac{\pi}{2}, 3\frac{\pi}{2}\right) &= \lim_{\eta \rightarrow 0} \lim_{\epsilon \rightarrow 0} q_{\eta, \epsilon}\left(\tau=0, \phi = \frac{\pi}{2}, 3\frac{\pi}{2}\right) \\ &= \lim_{\eta \rightarrow 0} \lim_{\epsilon \rightarrow 0} \frac{C_\eta}{2\pi^2 \eta} \left| \tan \frac{\epsilon}{2} \right| = 0. \end{aligned} \quad (D6)$$

(4) For  $\tau=0, \phi \neq \pi/2, 3\pi/2$  we obtain

$$\begin{aligned} q_{0,0}\left(\tau=0, \phi \neq \frac{\pi}{2}, 3\frac{\pi}{2}\right) &= \lim_{\eta \rightarrow 0} \lim_{\epsilon \rightarrow 0} q_{\eta, \epsilon}\left(\tau=0, \phi \neq \frac{\pi}{2}, 3\frac{\pi}{2}\right) \\ &= \lim_{\eta \rightarrow 0} \lim_{\epsilon \rightarrow 0} \frac{C_\eta}{2\pi^2 \eta} \frac{|1 + \cos \epsilon e^{2i\phi}|}{|\sin \epsilon|} \\ &\rightarrow \infty. \end{aligned} \quad (D7)$$

(5) Also, the function  $q_{0,0}(\tau, \phi)$  is normalized to unity:

$$\begin{aligned} \int_0^1 d\tau \int_0^{2\pi} q_{0,0}(\tau, \phi) d\phi &= \lim_{\eta, \epsilon \rightarrow 0} \int_0^1 d\tau \int_0^{2\pi} q_{\eta, \epsilon}(\tau, \phi) d\phi \\ &\times d\tau d\phi = 1. \end{aligned} \quad (D8)$$

These conditions define the function

$$q_{0,0}(\tau, \phi) = \delta(\tau) \frac{1}{2\pi}. \quad (D9)$$

We thus arrive at Eq. (4.26).

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